Four-dimensional aether-like Lorentz-breaking QED revisited and problem of ambiguities

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Abstract

In this paper, we consider the perturbative generation of the CPT-even aether-like Lorentz-breaking term in the extended Lorentz-breaking QED within different approaches and discuss its ambiguities.

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I. INTRODUCTION

The interest in the study of different aspects of the Lorentz symmetry breaking is very high now. Initially, it was motivated by the paper [1] where the first known example of the Lorentz-breaking extensions for the field theory models was presented, that is, the electrodynamics with the Lorentz-CPT breaking Carroll-Field-Jackiw (CFJ) term, which can be radiatively induced if a Lorentz and CPT violating axial term is included in the fermionic sector. Different results for the CFJ term have been obtained in a number of papers, see f.e. [2–4]. The key feature of this induction consists in its ambiguity, i.e. this term, arisen as a quantum correction, is finite, but the result for it depends on the procedure of calculation since the corresponding contribution is superficially divergent. This ambiguity was shown to be related with the axial anomaly [5], and it was shown in [6] that the effect of the completely undetermined value of the CFJ coefficient naturally emerges within the functional integral formalism. Further, this ambiguity was shown to take place also in the non-Abelian extension of the Lorentz-breaking QED [7], and in the finite temperature case [7, 8].

At the same time, the CFJ term is not the unique term displaying the ambiguity in the one-loop approximation. A similar situation takes place also for the four-dimensional gravitational Chern-Simons term [9] whose perturbative generation has been discussed in details in [10].

However, both CFJ term and gravitational Chern-Simons term break not only the Lorentz symmetry but also the CPT symmetry. Therefore, the natural question is whether the ambiguity takes place for the CPT-even terms, that is, those ones proportional to a constant even-rank tensor. In the recent papers [11, 12], the four-dimensional aether-like term in the nonmimimal QED, with a magnetic coupling only, was discussed and shown to be ambiguous (a number of issues related to this term has been discussed in [13–20]). Further, in [21] it was shown that this ambiguity vanishes, if we consider a gauge theory involving both minimal and nonminimal couplings, and impose a gauge-preserving regularization. However, the problem of ambiguity of the aether term is still open, even in the theory involving two couplings – it is worth mentioning that the three-derivative Myers-

Pospelov term was shown to be ambiguous being generated on the base of the theory with two couplings [22].

The aim of this paper consists in the study of the one-loop perturbative generation of the aether-like term in the four-dimensional Lorentz-breaking QED with two couplings and an axial term in the fermionic sector. We will show that the result for it is not exhausted by the contributions discussed in [11, 21], and, moreover, it displays the same ambiguity as the CFJ term.

II. GENERATION OF THE AETHER TERM FOR THE MASSIVE FERMIONS

To begin our study, we formulate the extended spinor electrodynamics which involves two couplings (the minimal one, proportional to e, and the nonminimal one, proportional to g) and an axial term in the fermionic sector:

$$\mathcal{L} = \bar{\psi}\left[i\partial\!\!\!/ - \gamma^\mu(eA_\mu + g\epsilon_{\mu\nu\lambda\rho}F^{\nu\lambda}b^\rho) - m - \gamma_5 b\!\!\!/\right]\psi. \tag{1}$$

Unlike the model considered in [11], this model can be reduced to the usual extended QED in the limit $g \to 0$. Namely this action has been considered in [21, 22]. Here the b_ρ is a constant vector implementing the Lorentz symmetry breaking, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the usual stress tensor constructed on the base of the gauge field A_μ.

Let us treat the gauge field as a purely external one, just as within the Schwinger approach. In this case the theory will be renormalizable. And its (one-loop) quantum correction in the effective action looks like

$$S_{eff}[b,A] \;=\; -i\,\mathrm{Tr}\,\ln(i\partial\!\!\!/ - e\gamma^\mu A_\mu - g\epsilon_{\mu\nu\lambda\rho}\gamma^\mu F^{\nu\lambda}b^\rho m - \gamma_5 b\!\!\!/), \tag{2}$$

The correction of the second order in the Lorentz-breaking vector b_μ in a purely nonminimal sector, where $e = 0$, has been discussed in details in [11, 21]. It was shown there that this correction looks like

$$S_{FF}(p) = -\frac{g^2}{2}\epsilon^{\alpha\beta\gamma\delta}\epsilon^{\alpha'\beta'\gamma'\delta'}b_\alpha F_{\beta\gamma}(p)b_{\alpha'}F_{\beta'\gamma'}(-p) \tag{3}$$
$$\times\int\frac{d^4k}{(2\pi)^4}\frac{1}{[k^2-m^2]^2}\mathrm{tr}[m^2\gamma_\delta\gamma_{\delta'} + k^\mu k^\nu\gamma_\mu\gamma_\delta\gamma_\nu\gamma_{\delta'}],$$

which can be rewritten as

$$S_{FF}(p) \;=\; C_0 g^2 m^2 (b^\alpha F_{\alpha\beta})^2, \tag{4}$$

where the constant C_0 is known to be equal either to $\frac{1}{4\pi^2}$ or to zero, see [11]. This is just the aether term proposed in [23]. It represents itself as a particular form of the most general CPT-even term $k^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}$ whose properties on the tree level have been studied in [17–20]. Further, in [21] it was shown that the value $C_0 = 0$ is preferable since the same constant arises during the calculation of the Proca-like correction, whose vanishing is natural from the viewpoint of the gauge invariance (note however, in principle, nobody forbids to use different values for the constant C_0 when Proca-like and aether-like terms are considered, since the constant C_0 is regularization dependent). Nevertheless, it is very important that within the model (1) there exists a much more powerful source of ambiguities. From now, we will discuss it.

So, let us turn to the aether-like corrections essentially depending on e. There are two such correction: one is purely minimal, proportional to e^2, and another one is nonminimal, proportional to eg.

First, the correction of the second order in the Lorentz-breaking vector b_μ in a purely minimal sector, where $g = 0$, is given by the following expression:

$$\begin{aligned} S_{AA}(p) \;=\; & \frac{ie^2}{2}\int\frac{d^4l}{(2\pi)^4}(\gamma^\mu\frac{1}{\not{l}-m}\gamma^\nu\frac{1}{\not{l}+\not{p}-m}\gamma_5\not{b}\frac{1}{\not{l}+\not{p}-m}\gamma_5\not{b}\frac{1}{\not{l}+\not{p}-m}+\\ & +\;\gamma^\mu\frac{1}{\not{l}-m}\gamma_5\not{b}\frac{1}{\not{l}-m}\gamma^\nu\frac{1}{\not{l}+\not{p}-m}\gamma_5\not{b}\frac{1}{\not{l}+\not{p}-m}+\\ & +\;\gamma^\mu\frac{1}{\not{l}-m}\gamma_5\not{b}\frac{1}{\not{l}-m}\gamma_5\not{b}\frac{1}{\not{l}-m}\gamma^\nu\frac{1}{\not{l}+\not{p}-m})A_\mu(-p)A_\nu(p), \end{aligned} \tag{5}$$

where p is an external momentum. This expression must be expanded up to the second order in p. A straightforward calculation along the lines similar to [22], after disregarding the irrelevant terms of third and higher order in derivatives and identically vanishing terms, yields

$$S_{AA} = -\frac{e^2}{6m^2\pi^2}b_\mu F^{\mu\nu}b^\lambda F_{\lambda\nu}. \tag{6}$$

We note that this expression is finite, ambiguity free (the last fact follows from the observation that the contribution of second order in p_μ obtained by expansion of (5) in series

of p is superficially finite) and gauge invariant. Indeed, we note that this reproduces the form of the aether term obtained in [11] through purely nonminimal interaction, while the numerical coefficient is naturally different.

Now, let us consider the "mixed" Feynman diagrams, involving both minimal and nonminimal couplings, and depicted at Fig.1.

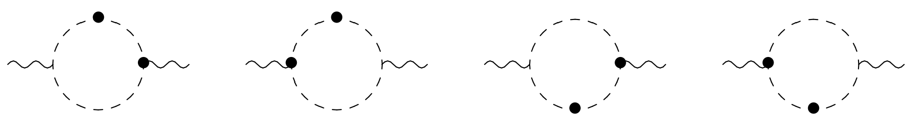

FIG. 1: Contributions to the two-point function of the vector field.

Similarly to the calculations in [22], here we consider the Lorentz-breaking insertions $\gamma_5 \not{b}$ introduced both in the vertices and in the propagators (both these insertions are denoted by the symbol $\bullet$). The calculations do not essentially differ from those ones carried out in [22], since the loop integrals and traces are identically the same (actually, the only difference from those papers is related to the fact that in one of the vertices, the A_μ field is replaced by its "dual" $\epsilon_{\mu\nu\lambda\rho}b^\nu F^{\lambda\rho}$. As a result, we arrive at

$$S_{AF} = eg \int d^4x \, k_\rho \epsilon^{\rho\nu\lambda\mu}(\epsilon_{\nu\alpha\beta\gamma}F^{\alpha\beta}b^\gamma \partial_\lambda A_\mu + A_\nu \epsilon_{\mu\kappa\eta\sigma}\partial_\lambda F^{\kappa\eta}b^\sigma), \tag{7}$$

where k_ρ is a constant vector whose explicit form is

$$k_\rho = 2i \int \frac{d^4l}{(2\pi)^4} \frac{b_\rho(l^2+3m^2) - 4l_\rho(b\cdot l)}{(l^2-m^2)^3}. \tag{8}$$

It is well known (see f.e. [7, 8]) that the integral (8) is ambiguous, looking like $k_\rho = Cb_\rho$, while a finite constant C crucially depends on the regularization prescription: within different procedures, it is equal to $\frac{1}{4\pi^2}$, $\frac{3}{8\pi^2}$, $\frac{3}{16\pi^2}$, zero, etc.

Multiplying the Levi-Civita symbols, we arrive at

$$S_{AF} = -2Cegb_\mu F^{\mu\nu}b^\lambda F_{\lambda\nu}. \tag{9}$$

So, we see that the ambiguity of the aether term arises from the "mixed" contribution involving both minimal and nonminimal couplings. Its form is similar to the usual ambiguity of the CFJ term (and of the higher-derivative terms [22]), and this ambiguity essentially

differs from the ambiguity of the aether term arisen from the purely nonminimal sector and discussed in [11, 21]. Therefore, we conclude that even the introduction the minimal interaction (as it has been done in [11, 21]) does not allow to rule out the ambiguity of the aether term.

The complete result for the one-loop two-point function of the gauge field A_μ is a sum of (4,6,9). It looks like

$$\begin{aligned}\Gamma_2 &= S_{AA} + S_{AF} + S_{FF} = \\ &= (C_0 g^2 m^2 - \frac{e^2}{6m^2\pi^2} - 2Ceg) b_\mu F^{\mu\nu} b^\lambda F_{\lambda\nu}.\end{aligned} \tag{10}$$

We conclude that this two-point function, first, displays the characteristic structure of the aether term [11], and, second, is finite and ambiguous. Moreover, it involves two ambiguous constants C_0 and C, and, while the regularization where $C_0 = 0$, is preferable, in order to cancel the undesirable Proca term (see discussion in [21]), there is no any preferable regularization allowing to fix the value of C.

III. MASSLESS FERMIONS CASE

In the previous section, we have considered the massive fermions case. However, it is instructive to do the calculations also for the massless fermions where they are much simpler. Just for comparison, we use two approaches.

A. Functional calculation

In the the massless case, the fermion action is given by

$$\Sigma_\psi = \int d^4x \ \bar{\psi}(i\partial\!\!\!/ - \tilde{A}\!\!\!/ - b\!\!\!/\gamma_5)\psi, \tag{11}$$

which is the action (1) with $m = 0$, where we used the redefined gauge field

$$\tilde{A}_\mu = eA_\mu + g\varepsilon_{\mu\nu\alpha\beta} b^\nu F^{\alpha\beta}. \tag{12}$$

It is instructive to discuss first a nonperturbative calculation (in b and in the coupling constant) of the induced Lorentz violating terms. This calculation is similar to the one

performed in [6]. By making the chiral transformation,

$$\psi \to e^{-i\gamma_5 b\cdot x}\psi \ , \ \bar{\psi} \to \bar{\psi} e^{-i\gamma_5 b\cdot x}, \tag{13}$$

we can eliminate the b_μ vector from the classical action. Nevertheless, repeating the arguments from [24], one can show that, at the quantum level, the measure of the generating functional acquires a factor given by the following Jacobian:

$$J[b_\mu, \tilde{A}_\mu] = exp\left\{ -i \int d^4x \ (b\cdot x)\mathcal{A}[\tilde{A}_\mu](x) \right\}, \tag{14}$$

with

$$\mathcal{A}[\tilde{A}_\mu](x) = \frac{1}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}\tilde{F}_{\mu\nu}\tilde{F}_{\alpha\beta}, \tag{15}$$

where $\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$. We can write

$$J[b_\mu, \tilde{A}_\mu] = exp\left\{ -i \int d^4x \ \frac{1}{4\pi^2}(b\cdot x)\epsilon^{\mu\nu\alpha\beta}\partial_\mu\tilde{A}_\nu\partial_\alpha\tilde{A}_\beta \right\}, \tag{16}$$

which after an integration by parts turns out to be

$$J[b_\mu, \tilde{A}_\mu] = exp\left\{ i \int d^4x \ \frac{1}{4\pi^2}b_\mu\epsilon^{\mu\nu\alpha\beta}\tilde{A}_\nu\partial_\alpha\tilde{A}_\beta \right\}. \tag{17}$$

We see that after the chiral transformation the axial term disappears from the fermionic sector. As a result the QED Lagrangian is obtained, together with a Jacobian which is taken into account when quantum corrections are calculated.

We now can return to the usual gauge field and obtain the complete induced Lorentz violating terms. The induced Lagrange density is given by

$$\mathcal{L}_{ind} = \mathcal{L}_b + \mathcal{L}_{bb} + \mathcal{L}_{bbb}, \tag{18}$$

with

$$\mathcal{L}_b = \frac{1}{4\pi^2}e^2 b_\mu \varepsilon^{\mu\nu\alpha\beta}A_\nu\partial_\alpha A_\beta, \tag{19}$$

$$\begin{aligned}\mathcal{L}_{bb} &= \frac{1}{4\pi^2}egb_\mu\left(b^\theta\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\beta\theta\delta\tau}A_\nu\partial_\alpha F^{\delta\tau} + b^\rho\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\nu\rho\sigma\lambda}\partial_\alpha A_\beta F^{\sigma\lambda}\right) \\ &= \frac{1}{4\pi^2}eg\left(-b^2F_{\alpha\beta}F^{\alpha\beta} + 4(b_\mu F^{\mu\alpha})^2\right)\end{aligned} \tag{20}$$

and

$$\mathcal{L}_{bbb} = \frac{1}{4\pi^2} g^2 b_\mu b^\rho b^\theta F^{\sigma\lambda}\partial_\alpha F^{\delta\tau}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\nu\rho\sigma\lambda}\varepsilon_{\beta\theta\delta\tau}$$
$$= -\frac{1}{4\pi^2} g^2 \left(2b^2 b^\theta F^{\alpha\beta}\partial_\alpha F^{\delta\tau}\varepsilon_{\beta\theta\delta\tau} - 2b_\mu b^\alpha b^\theta F^{\mu\beta}\partial_\alpha F^{\delta\tau}\varepsilon_{\beta\theta\delta\tau}\right). \tag{21}$$

The three terms above give the total contribution to the quantum corrections to the photon sector arisen due to the axial term. The first one is simply the induced Chern-Simons term which has been discussed in the last decade (see f.e. [2]). The second one contains the aether term and the rescaled Maxwell term. The third one is composed by higher derivative terms. Their different aspects have been considered in [22, 26, 28]. These coefficients are actually ambiguous, since there is a freedom in the definition of the chiral current. This ambiguity, as we will see, manifests itself as as a regularization dependence in the perturbative calculation.

B. The massless complete one-loop calculation

We now carry out an one-loop analysis for the massless case. Using the modified gauge field $\tilde{A}_\mu$, this is an easy task, since the complete one-loop amplitude can be written in terms of the vacuum polarization tensor of the modified massless QED, with only the axial term. In principle, one can use each of the many results obtained by great number of regularization techniques. We use the one of [25], since its expression in function of surface terms allows us to discuss different possibilities and to identify the regularization dependence of the induced terms. The following result has been established in [25]:

$$T^{\mu\nu} = T_0^{\mu\nu} + T_b^{\mu\nu} + T_{bb}^{\mu\nu}, \tag{22}$$

with

$$T_0^{\mu\nu} = \Pi(p^2)(p^\mu p^\nu - p^2 g^{\mu\nu}) - 4\alpha_1 g^{\mu\nu} - \frac{4}{3}\left[\alpha_2(p^\mu p^\nu - p^2 g^{\mu\nu})\right.$$
$$\left. + (2p^\mu p^\nu + p^2 g^{\mu\nu})(\alpha_3 - 2\alpha_2)\right], \tag{23}$$

$$T_b^{\mu\nu} = -4i\alpha_2 p_\alpha b_\beta \epsilon^{\mu\nu\alpha\beta} \tag{24}$$

and

$$T_{bb}^{\mu\nu} = -4\left\{\left(b^2 g^{\mu\nu} + 2b^\mu b^\nu\right)(\alpha_3 - 2\alpha_2)\right\}, \tag{25}$$

where the coefficients $\alpha_1, \alpha_2, \alpha_3$ are introduced in [25]. Their explicit form is

$$\begin{aligned}
\alpha_1 g_{\mu\nu} &= \int \frac{d^4k}{(2\pi)^4}\frac{\partial}{\partial k^\mu}\frac{k_\nu}{k^2-\lambda^2};\\
\alpha_2 g_{\mu\nu} &= \int \frac{d^4k}{(2\pi)^4}\frac{\partial}{\partial k^\mu}\frac{k_\nu}{(k^2-\lambda^2)^2};\\
\alpha_3 g_{\{\mu\nu}g_{\alpha\beta\}} &= \int^\Lambda \frac{d^4k}{(2\pi)^4}\frac{\partial}{\partial k^\beta}\left[\frac{4k_\mu k_\nu k_\alpha}{(k^2-\lambda^2)^3}\right] + \alpha_2 g_{\{\mu\nu}g_{\alpha\beta\}},
\end{aligned} \tag{26}$$

where $g_{\{\mu\nu}g_{\alpha\beta\}}$ is a symmetrized product of two Minkowski metrics and λ is a mass scale.

Let us argue that the term relevant for our purpose is that one linear in b (24). In principle, the complete one-loop photon self-energy for the present model can involve fourth order in b, because of the quadratic part $T_{bb}^{\mu\nu}$ and the two additional b arising from the nonminimal vertices. However, a simple observation of the equations (23) and (25) shows that the condition of transversality of $T^{\mu\nu}$ imposes the surface terms to respect the relations $\alpha_1 = 0$ and $\alpha_3 = 2\alpha_2$. These conditions set $T_{bb}^{\mu\nu} = 0$. So the higher order term is of the order b^3.

Let us also discuss the contribution to the Lorentz violating part coming from $T_0^{\mu\nu}$. In [21], it was shown that, since we are interested in the limit $p^2 \to 0$, the contributions to the CPT odd and even terms coming from $T_0^{\mu\nu}$ have the fermion mass in their coefficients. So, for this massless case, they do not contribute. We are then left with $T_b^{\mu\nu}$.

We have now a simple task, if we use the redefined field $\tilde{A}_\mu$ of equation (12). We can write the induced term as

$$\mathcal{L}_{LV} = -2i\alpha_2 b_\mu \varepsilon^{\mu\nu\alpha\beta}\tilde{A}_\nu \partial_\alpha \tilde{A}_\beta. \tag{27}$$

With a simple substitution in terms of the A_μ field we obtain the complete one loop Lorentz violating induced term:

$$\begin{aligned}
\mathcal{L}_{LV} = &-2i\alpha_2 e^2 b_\mu \varepsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta - 2i\alpha_2 eg\left(-b^2 F_{\alpha\beta}F^{\alpha\beta} + 4(b_\mu F^{\mu\alpha})^2\right)\\
&-2i\alpha_2 g^2\left(2b^2 b^\theta F^{\alpha\beta}\partial_\alpha F^{\delta\tau}\varepsilon_{\beta\theta\delta\tau} - 2b_\mu b^\alpha b^\theta F^{\mu\beta}\partial_\alpha F^{\delta\tau}\varepsilon_{\beta\theta\delta\tau}\right).
\end{aligned} \tag{28}$$

It is worth to note that all these terms have the surface term α_2 as their coefficient. This surface term cannot be fixed in gauge invariance grounds, the unique condition being $\alpha_3 = 2\alpha_2$. This fact reveals an unavoidable ambiguity in the coefficients of the Lorentz violating induced terms. This differs from the case studied in [21], where the axial term was absent, and the coefficient of the induced terms, that is, α_1, should be fixed to be equal to zero in order to preserve the transversality of the vacuum polarization tensor of the traditional QED sector. Particularly, the aether-like contribution is essentially ambiguous.

IV. SUMMARY

Now, let us discuss our results. We have considered the perturbative generation of the aether-like term in the extended Lorentz-breaking QED whose action involves both couplings, the minimal one and the nonminimal one [21] and, besides, an axial term in the fermionic sector. We have found that this term is exactly the same one considered in [11], being gauge invariant and UV finite despite the superficial logarithmic divergence of the corresponding contribution. We note that, besides the ambiguity of the aether term discussed earlier in [11, 21] and characterized by the coefficient C_0, a new ambiguity described by the coefficient C, which is identically the same as that one accompanying the CFJ term in the usual Lorentz-breaking QED [7, 8], also arises. This shows that the ABJ anomaly which is known to be responsible for the ambiguity of the CFJ term [5] can be naturally promoted to a wide class of new terms. One must note, however, that this ambiguity disappears if we switch off the nonminimal interaction. Therefore, it does not arise in the usual extended QED, although the aether-like term arises also in this case. Moreover, differently from the ambiguity of the aether term considered in [11, 21], the new ambiguity cannot be removed via the choice of a gauge-preserving regularization. Also, we note that the functional integral approach developed in [6] and applied here for the massless fermions, can be naturally generalized for the massive fermions case, whereas this calculation seems to be very complicated from the technical viewpoint. We are planning to do it in a forthcoming paper.

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